Section 4.4 Related Rates (Minimum Homework: 1, 3, 5, 7)

1) A pebble is dropped into a calm pond causing ripples in the form of concentric circles. The radius *r* of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area of the disturbed water changing?

1. Identify the information that is given.

Let $r = outer\ radius$

$\frac{dr}{dt} = rate\ of\ change\ the\ radius\ is\ growing\ in\ feet\ per\ second$

Given:
$\frac{dr}{dt} = 1\ \ foot\ per\ second$

$r = 4\ ft$

2. Identify what needs to be solved for.

$We\ are\ asked\ to\ find\ the\ rate\ at\ which\ the\ total\ area\ of\ the\ disturbed\ water\ is\ changing.$

Symbolically, we are asked to find $\frac{dA}{dt}$ where A represents the disturbed area.

3. Write an equation that relates the variables. To derive the equation you may use a geometric fact (like an area or a volume formula)

$We\ need\ the\ area\ formula\ for\ a\ circle$:

$A = \pi r^2$

4. Take the derivative $\frac{d}{dt}$ of both sides of the equation.

$\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2)$

$1\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

5. Solve for the unknown rate of change.

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

6. Substitute all known values to get the final answer.

Substitute $r = 4 \ and \frac{dr}{dt} = 1$

$\frac{dA}{dt} = 2\pi(4)(1)$

$\frac{dA}{dt} = 8\pi$

Now write an answer with units.

Answer: The disturbed area is growing at a rate of $8\pi \ feet\ squared\ per\ second$

3) A pebble is dropped into a calm pond causing ripples in the form of concentric circles. The radius *r* of the outer ripple is increasing at a constant rate of 3 feet per second. When the radius is 5 feet, at what rate is the total area of the disturbed water changing?

1. Identify the information that is given.

Let $r = outer\ radius$

$\frac{dr}{dt} = rate\ of\ change\ the\ radius\ is\ growing\ in\ feet\ per\ second$

Given:
$\frac{dr}{dt} = 3\ \ feet\ per\ second$

$r = 5\ ft$

2. Identify what needs to be solved for.

$We\ are\ asked\ to\ find\ the\ rate\ at\ which\ the\ total\ area\ of\ the\ disturbed\ water\ is\ changing.$

Symbolically, we are asked to find $\frac{dA}{dt}$ where A represents the disturbed area.

3. Write an equation that relates the variables. To derive the equation you may use a geometric fact (like an area or a volume formula)

$We\ need\ the\ area\ formula\ for\ a\ circle$:

$A = \pi r^2$

4. Take the derivative $\frac{d}{dt}$ of both sides of the equation.

$\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2)$

$1\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

5. Solve for the unknown rate of change.

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

6. Substitute all known values to get the final answer.

Substitute $r = 5 \ and \frac{dr}{dt} = 3$

$\frac{dA}{dt} = 2\pi(5)(3)$

$\frac{dA}{dt} = 30\pi$

Now write an answer with units.

Answer: The disturbed area is growing at a rate of $30\pi \ feet \ squared \ per \ second$

5) Air is being pumped into a spherical balloon at 10 cm^3/minute. Calculate the rate at which the radius of the balloon is changing when the radius of the balloon is 6 cm.

1. Identify the information that is given.

   Let $r = radius\ of\ balloon$
   $V = volume \quad (amount\ of\ air\ inside\ the\ baloon)$

Given: $r = 6\ cm$

Given: $\frac{dV}{dt} = 10\ cm^3 per\ second$

2. Identify what needs to be solved for.

We are asked to find how fast the radius is changing.

That is we are asked to find $\frac{dr}{dt}$ (hint units of our answer will be cm/sec as the length of the radius is given in centimeters, and time is given in seconds)

3. Write an equation that relates the variables. To derive the equation you may use a geometric fact (like an area or a volume formula)

   We need the volume formula for a sphere.

   $V = \frac{4}{3}\pi r^3$

4. Take the derivative $\frac{d}{dt}$ of both sides of the equation.

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r\text{^}3\right)$

$1 * \frac{dV}{dt} = \left(3 * \frac{4}{3}\pi r^2\right)\frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

5. Solve for the unknown rate of change. $we\ need\ to\ solve\ for \frac{dr}{dt}$
   $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

Divide both sides by $4\pi r^2$

$$\frac{\frac{dV}{dt}}{4\pi r^2} = dr/dt$$

$$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{4\pi r^2}$$

6. Substitute all known values to get the final answer.

Let $\frac{dV}{dt} = 10 \quad and \quad r = 6$

$$\frac{dr}{dt} = \frac{10}{4\pi(6)^2} = \frac{10}{144\pi} = \frac{5}{72\pi}$$

The units in my answer should be centimeters per second, as the radius is measured in centimeters and the time is given in seconds.

Answer: Radius is growing at a rate of $\frac{5}{72\pi}\, centimeters\ per\ second$

7) Air is being pumped into a spherical balloon at 3 cm^3/minute. Calculate the rate at which the radius of the balloon is changing when the radius of the balloon is 8 cm.

1. Identify the information that is given.

   Let $r = radius\ of\ balloon$
   $V = volume\quad (amount\ of\ air\ inside\ the\ baloon)$

Given: $r = 8\ cm$

Given: $\frac{dV}{dt} = 3\ cm^3 per\ second$

2. Identify what needs to be solved for.

We are asked to find how fast the radius is changing.

That is we are asked to find $\frac{dr}{dt}$ (hint units of our answer will be cm/sec as the length of the radius is given in centimeters, and time is given in seconds)

3. Write an equation that relates the variables. To derive the equation you may use a geometric fact (like an area or a volume formula)

   We need the volume formula for a sphere.

   $V = \frac{4}{3}\pi r^3$

4. Take the derivative $\frac{d}{dt}$ of both sides of the equation.

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r\text{^}3\right)$

$1 * \frac{dV}{dt} = \left(3 * \frac{4}{3}\pi r^2\right)\frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

5. Solve for the unknown rate of change. $we\ need\ to\ solve\ for \frac{dr}{dt}$
   $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

Divide both sides by $4\pi r^2$

$$\frac{\frac{dV}{dt}}{4\pi r^2} = dr/dt$$

$$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{4\pi r^2}$$

6. Substitute all known values to get the final answer.

Let $\frac{dV}{dt} = 3 \quad and \quad r = 8$

$$\frac{dr}{dt} = \frac{3}{4\pi(8)^2} = \frac{3}{4*64\pi} = \frac{3}{256\pi}$$

The units in my answer should be centimeters per second, as the radius is measured in centimeters and the time is given in seconds.

Answer: Radius is growing at a rate of $\frac{3}{256\pi} centimeters\ per\ second$